Experimental Design for Cost-Aware Learning of Causal Graphs

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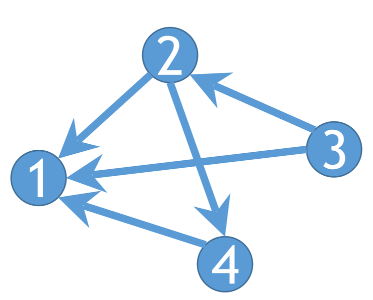
Causal Graphs

We assume there is some underlying *causal graph* $D = (V, E)$. *If x causes y, then there exists a directed edge from x to y.*

This is Pearl's structural causal model (Pearl '09).



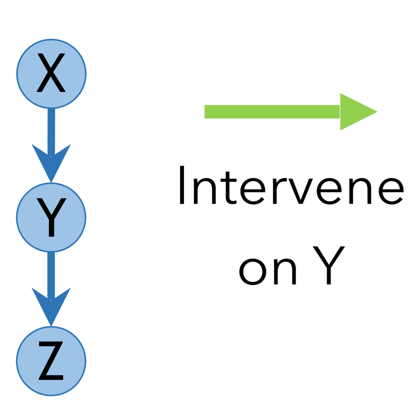
$$x_1 = f_1(x_2, x_3, x_4, e_1)$$
$$x_2 = f_2(x_3, e_2)$$
$$x_3 = f_3(e_3)$$
$$x_4 = f_4(x_2, e_4)$$

e_i are jointly independent.

Interventions

An intervention is an experiment which fixes the value of a set of random variables.

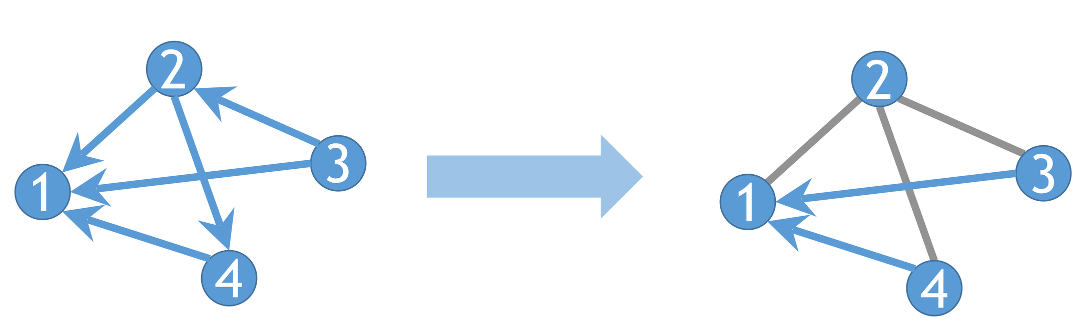
Intervention changes the causal graph. Use conditional independence tests to detect the changes.



Learning From Data

From conditional independence tests, we can learn:

- *graph skeleton* (undirected version)
- *v-structures* $(a \to b \leftarrow c)$

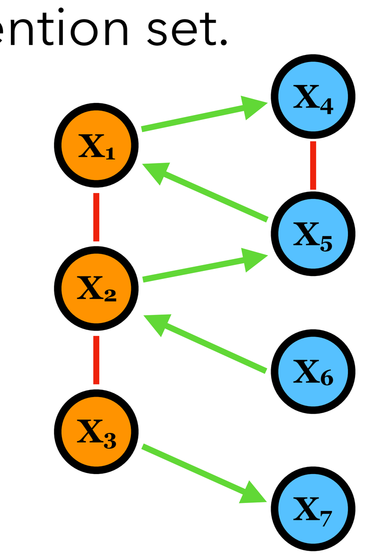


Meek's rules learn other edges by using acyclicity and other properties (Meek '95).

Remaining undirected graph is *chordal*.

Learning From Interventions

We can learn all the edges in the cut defined by the intervention set.



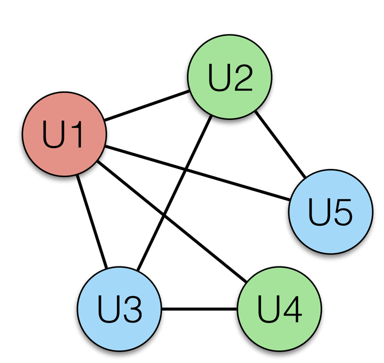
Each intervention may have a different cost.

Intervention Designs

We consider *non-adaptive* case: all experiments must be designed before seeing results.

Need to choose a set of graph cuts such that all edges are covered by some cut. This is called a *graph separating system*.

Every graph separating system corresponds to some proper coloring of the graph (Hauser and Bühlmann '12).



	I_1	I_2
U1	1	0
U2	0	1
U3	1	1
U4	0	0
U5	0	0

	I_1	I_2
U1	1	0
U2	0	1
U3	0	0
U4	0	1
U5	0	0

	I_1	I_2
Red	1	0
Gre	0	1
Blue	0	0

Graph separating systems

Minimum Cost Intervention Design

Suppose intervening on a vertex v has a cost w_v. First considered by (Kocaoglu et al. '17), the cost of an intervention design $\mathcal{I}$ is

$$\text{cost}(\mathcal{I}) = \sum_{I \in \mathcal{I}} \sum_{v \in I} w_v.$$

Problem: Given a chordal graph, find an intervention design with at most m interventions of minimal cost.

It is equivalent to a weighted coloring problem where there are $\binom{m}{k}$ colors of weight $k = 0, 1, \ldots, m$.

Hardness Result

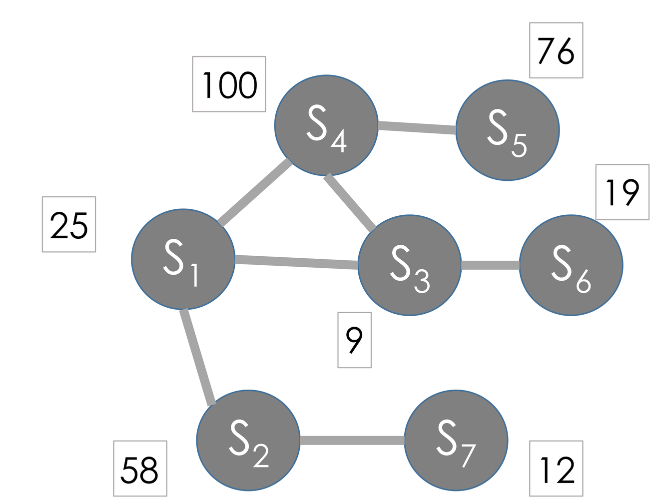
Theorem: The minimum cost intervention design problem is NP-hard for arbitrary chordal graphs, even in the unweighted case. [Reduction from Numerical 3D Matching.]

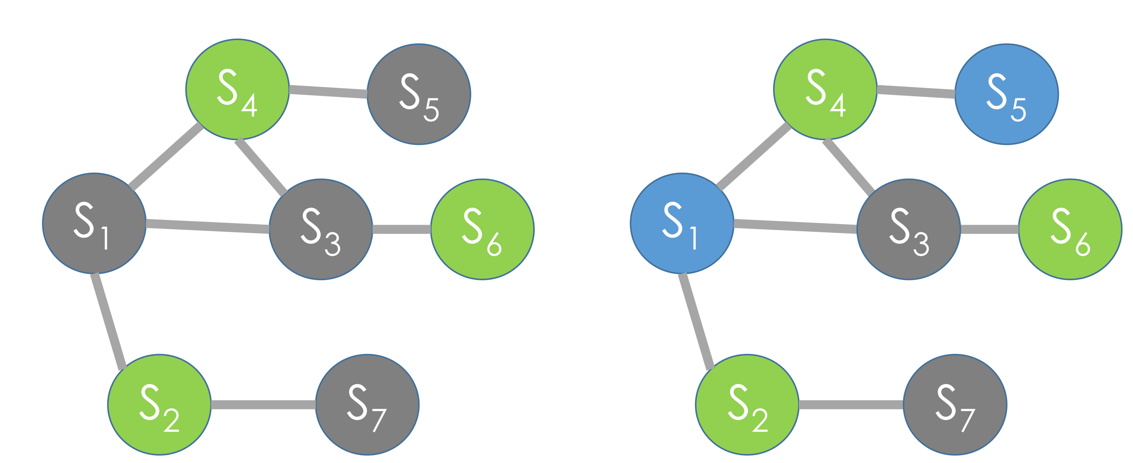
A Greedy Algorithm

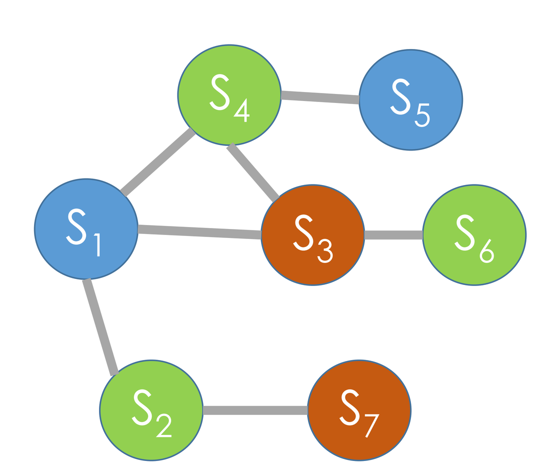
Input is chordal, can find the maximum weighted independent set in polynomial time.

Greedy Algorithm (Kocaoglu et al. '17):

- While the graph is not fully colored:
  - Find the max weight independent set
  - Color this set with the next cheapest color
  - Remove this set from the graph







S_1	1	0	0
S_2	0	0	0
S_3	0	1	0
S_4	0	0	0
S_5	1	0	0
S_6	0	0	0
S_7	0	1	0

However the number of steps needed or approximation guarantees were unknown.

Approximation Guarantees

Quantize the vertex weights not in the maximum independent set so that they are all bounded by a polynomial, then run the greedy algorithm.

Theorem: If the number of interventions m satisfies

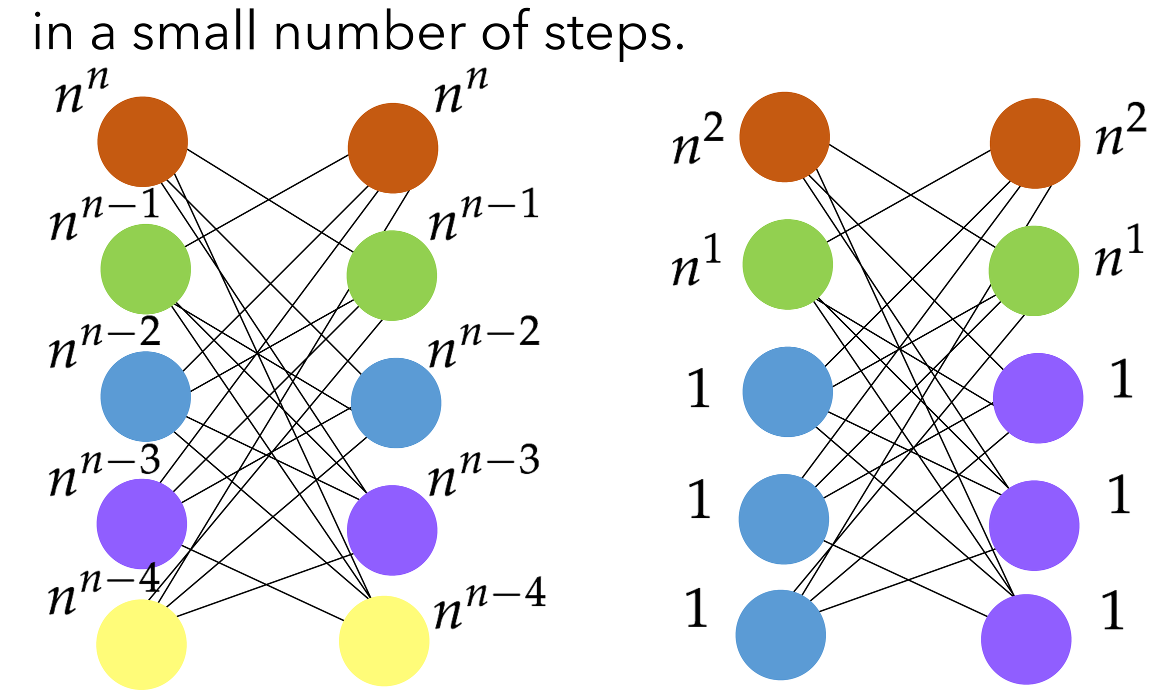
$$m \geq \log \chi + O(\log \log n),$$

then the greedy algorithm is a factor of $(2 + \varepsilon)$ from the optimal cost.

Proof Idea:

Correctness: Show the greedy algorithm is also the greedy algorithm for a particular submodular set cover problem (Wolsey '82).

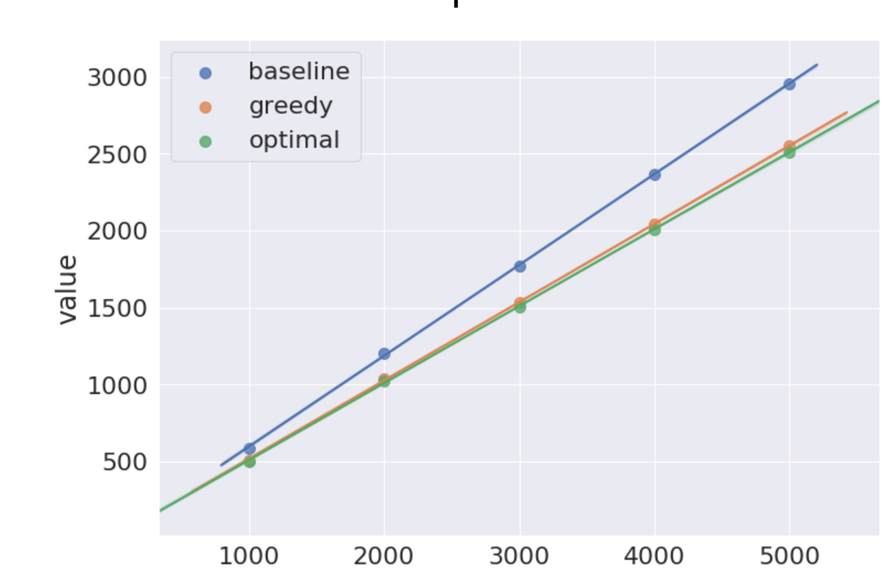
Quantization allows the algorithm to terminate in a small number of steps.



Approximation: Uses results from submodular optimization.

Experiments

We generate random chordal graphs of various sizes using the approach of Shanmugam et al., (NeurIPS 2015). We generate costs using a heavy-tailed distribution. We compare the solution value to the optimal solution.



We see that the greedy algorithm performance is nearly optimal.